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1. Statement of the Problems Studied

The research on simultaneous inference and ranking and selection procedures is important and relevant in comparing several populations (products, alternatives) in terms of their intrinsic quality or worth. This report embodies the research accomplishments in this broad area. The main contributions deal with newly developed ranking, selection and testing procedures based on Bayes and empirical Bayes approach. During the period April 1995 to September 2000, twenty-five research papers were completed by the PI and collaborators. Of these fifteen have been published and or accepted for publication in refereed journals and refereed conference proceedings volumes. The problems studied deal with a wide range of statistical models such as normal, Bernoulli, Poisson, and logistic distributions. In other papers, the statistical models are quite general in that the distributions are not specified but may belong to a broad family such as the positive or the general exponential family of distributions. One may want to know how good the empirical Bayes procedures are. This question is answered in terms of the convergence rate of the regret risk associated with empirical Bayes procedures. In general, it is found that the rate is optimal or very close to the optimal, where the optimal rate is the best achevable rate under certain conditions.

2. List of Manuscripts

2.1. Published and Accepted for Publicaiton in Refereed Journals.

During the period of April 1995 to September 2000, the following papers have been published and/or accepted for publication.

[1] Gupta, S. S. and Miescke, K. J. (1996). Bayesian look-ahead sampling allocations for selecting the best Bernoulli population. In *Research Developments in Probability and Statistics: Festschrift in Honor of Madan L. Puri* (Eds. E. Brunner and M. Denker), VSP International Science Publishers, Zeist, The Netherlands, 353-369.

[2] Gupta, S. S. and Panchapakesan S.(1996). Design of experiments with selection

and ranking goals. *Handbook of Statistics* (Eds. S. Ghosh and C. R. Rao), Elsevier Science B. V., Vol. 13, 555-584.

[3] Gupta, S. S. and Miescke, K. J.(1996). Bayesian look ahead one-stage sampling allocations for selection of the best population. *J. Statist. Plann. Inf.* Vol. 54, 229-244.

[4] Liang, T. (1997). Simultaneously Selecting Normal Populations Close to a Control. *J. Statist. Plan. Inf.* Vol. 61, 297-316.

[5] Gupta, S. S. and Liang, T. (1998). Simultaneous lower confidence bounds for probabilities of correct selections. *J. Statist. Plan. Inf.* Vol. 72 (1998), 279-290.

[6] Gupta, S. S. and Liang, T.(1999). On empirical Bayes simultaneous selection procedures for comparing normal populations with a standard. *J. Statist. Plann. Inf.* Vol. 77, 73-88.

[7] Gupta, S. S. and Liang, T. (1999). Selecting good exponential populations compared with a control : A nonparametric Bayes approach. *Sankhya, Ser. B.* Vol. 61, 289-304.

[8] Gupta, S. S. and Liang, T. (1999). On simultaneous selection of good populations. *Statistics & Decision.* Supplement Issue No. 4, 33-53

[9] Miescke, K. J. (1999) Bayes sampling designs for selection procedures. *Multivariate analysis, design of experiments, and survey sampling,* Statist. Textbooks Monogr., 159, Marcel Dekker, New York, 93-117.

[10] Liang, T.(1999). Empirical Bayes two-tail tests in a discrete exponential family. *Statistics & Decisions.* Vol. 17, 157-184.

[11] Gupta, S. S. and Liang, T. (2000). Selecting the most reliable Poisson population provided it is better than a control: a nonparametric empirical Bayes approach. To appear in the *J. Statist. Plann. Inf.*, special volume in honor of C. R. Rao's 80th Birthday.

[12] Gupta, S. S. and Miescke, K. J. (2000). On the performance of subset selection procedures under normality. To appear in the *J. Statist. Plann. Inf.*, special volume in honor of C. R. Rao's 80th Birthday.

[13] Gupta, S. S. Lin, Z. and Lin, X. (2000). On a selection procedure for selecting the best logistic population compared with a control. To appear in *Advances on Theoretical and Methodological Aspect of Probability and Statistics* (Ed. N. Balakrishnan), Gordon and Breach, Research Science Publishes, 345-370.

[14] Gupta, S. S. and Liese, F. (2000). Asymptotic distribution of the random regret risk for selecting exponential populations. To appear in *Kybernetika.*

[15] Li, J. and Gupta, S. S. (2000). Monotone empirical Bayes tests with optimal rate of convergence for a truncation parameter. To appear in *Statistics & Decisions.*

2.2. Submitted to Refereed Journals.

During the above period, the following research papers have been completed and submitted for possible publication.

[16] Gupta, S. S., He, S. and Li, J. (1999). On selection procedures for exponential family distributions based on type-I censored data. Department of Statistics, Purdue University, Technical Report # 99-06.

[17] Li, J. (2000). Optimal rate of monotone empirical Bayes tests for normal means.

2.3. Published as Technical Reports.

During the same above period, the following research papers have also been done and are available as Purdue Statistics Department Technical Reports. These papers will be submitted for publication in the near future.

[18] Liang, T.(1996). On empirical Bayes two-stage test in a discrete exponential family. Department of Statistics, Purdue University, Technical Report #96-31C.

[19] Gupta, S. S., Liang, T. and Lin, X. (1997). Empirical Bayes two-stage selection procedures for selecting the best Bernoulli treatment using inverse binomial sampling. Department of Statistics, Purdue University, Technical Report #97-16C.

[20] Gupta, S. S. and Lin, X. (1997). A selection problem in measurement error

models. Department of Statistics, Purdue University, Technical Report #97-18C.

[21] Gupta, S. S. and Li, J. (1999). Empirical Bayes tests with $n^{-1+\epsilon}$ convergence rate in continuous one-parameter exponential family. Department of Statistics, Purdue University, Technical Report # 99-09C.

[22] Gupta, S. S. and Li, J. (1999). Empirical Bayes tests for some non-exponential distribution family. Department of Statistics, Purdue University, Technical Report # 99-17.

[23] Gupta, S. S. and Li, J. (1999). Empirical Bayes tests in some continuous exponential family. Department of Statistics, Purdue University, Technical Report # 99-21.

[24] Gupta, S. S. and Li, J. (1999). An empirical Bayes procedure for selecting good populations in some positive exponential family. Department of Statistics, Purdue University, Technical Report # 99-25.

[25] Gupta, S. S. and Li, J. (2000). Optimal rate of convergence of monotone empirical Bayes tests for lower truncation parameters. Department of Statistics, Purdue University, Technical Report # 00-07.

3. Summary of the most important results.

Resaerch accomplishments have been made in the following areas.

3.1. Multiple Decision Theory with Special Reference to Selection and Ranking Procedures in simultaneous Inference.

Papers [1, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 16, 24] deal with the above problem in general. Most of the research is based on Bayes and empirical Bayes approach. The main results are as follows:

In [1], Gupta and Miescke studied the problem of selecting the best from among k independent Bernoulli populations using the Bayesian approach. Assume that k independent samples of sizes $n_1, \ldots, n_k$, respectively, have been observed already at a first stage, and that m more observations are allowed to be taken at a future second stage. The problem considered is how to allocate these m observations in a suitable manner among the k populations given the information gathered so far. Several allocation schemes were examined by Gupta and Miescke and compared analytically as well as numerically. A simple look-ahead one-observation-at-a-time allocation rule has been shown to have good performance properties.

In [3], Gupta and Miescke have investigated the selection problems for k independent populations from the underlying exponential family. Suppose one wants to find that population which has the highest parameter value, using a Bayes selection rule which is based on a known prior density and a given loss function. Assume that k independent samples of sizes $n_1, n_2, \ldots, n_k$, respectively, have been observed already at a first stage, and that m additional observations are planned to be taken at a second stage. The problem considered is how to allocate these m observations in a suitable manner among the k populations, given the information gathered so far. Several allocation schemes have been examined and compared analytically, as well as numerically. A simple look-ahead-one-observation-at-a-time allocation rule has been shown to have good performance properties. This paper studies, in detail, the case where the underlying distributions are normal.

Liang [4] has derived selection procedure for simultaneously selecting good populations compared with a control from among k normal populations. Liang [4] also investigated the asymptotic optimality and established the associated rate of convergence for the procedure. It is shown that the relative regret Bayes risk of the proposed empirical Bayes selection rule converges to zero with a rate of order $O(k^{-1})$.

Gupta and Liang [6] derived statistical selection procedures to partition k normal populations into "good" and "bad" ones, respectively, using the nonparametric empirical Bayes approach. The relative regret Bayes risk of a selection procedure is used as a measure of its performance. Gupta and Liang establish the asymptotic optimality of the proposed empirical Bayes selection procedure and investigate its associated rates of convergence. It is shown that under certain conditions the empirical Bayes selection procedures have rates of convergence of order $O(k^{-a(r-1)/(2r+1)})$, where $1 < a < 2$ and r

is an integer greater than 2.

In [7], Gupta and Liang investigated empirical Bayes procedures for selecting good exponential populations compared with a control. Based on the accumulated historical data, an empirical Bayes selection procedure is constructed by mimicking the behavior of a Bayes selection procedure. The empirical Bayes selection procedure is proved to be asymptotically optimal. The analysis shows that the rate of convergence of the selection rules is influenced by the tail probabilities of the underlying distributions. It is shown that under certain regularity conditions on the moments of the prior distribution, the empirical Bayes procedure is asymptotically optimal of order $O(n^{-\lambda/2})$ for some $0 < \lambda \leq 2$. A lower bound with rate of convergence of order $O(n^{-1})$ is also established for the regret Bayes risk of the procedure.

Gupta and Liang [8] considered the problem of simultaneous selection of k populations in comparison with a control. Under the assumption of linearity of the posterior mean, Gupta and Liang have derived an empirical Bayes selection procedure for selecting all good populations. They have also investigated the asymptotic optimality and rate of convergence of the proposed empirical Bayes procedure. It was proved that the rate of convergence for the empirical Bayes procedure is of the order $O(k^{-1})$.

Miescke [9] investigated Bayes sampling designs for selection procedures. From k independent populations which belong to a one-parameter exponential family $\{F_\theta\}$, $\theta \in \Omega \subset R$, random samples of sizes $m_1, \cdots, m_k$, respectively, are to be drawn. After the observations have been drawn, a selection procedure is used to determine which of these k populations has the largest value of θ. Given a prior for the k parameters, a Bayes selection procedure can be found and its Bayes risk can be determined, where both depend on $m_1, \cdots, m_k$. Let the sample size be restricted by $m_1 + \cdot + m_k = m$, where m is fixed. The problem of how to find the optimum (minimum Bayes risk) sample design subject to this constraint is considered, as well as m-truncated sequential sampling allocations. Results for normal and binomial families, under 0-1 loss and the linear loss, are presented and discussed. An introduction to Bayes selection procedures is included.

In [11], the problem of selecting the most reliable Poisson population from among k competitor provided it is better than a control was studied using the empirical Bayes approach. An empirical Bayes selection procedure was constructed based on the isotonic regression estimators of the posterior means of failure rates associated with the k Poisson populations. The asymptotic optimality of the empirical Bayes selection procedure was investigated. It has been shown that, under certain regularity conditions, the proposed empirical Bayes selection procedure is asymptotically optimal and the associated Bayes risk converges to the minimum Bayes risk at a rate of order $O(\exp(-cn))$ fro some $c > 0$, where n denotes the number of historical data at hand when the present selection problem is considered.

Gupta and Miescke [12] investigated the performance of subset selection procedures under normality. Suppose there are k normal populations $N(\theta_1, \sigma_1^2), \ldots, N(\theta_k, \sigma_k^2)$, where the means $\theta_1, \ldots, \theta_k$ are unknown, and the variances $\sigma_1^2, \ldots, \sigma_k^2$ are known. Independent random samples of size $n_1, \ldots, n_k$ are drawn from each normal population respectively. Based on these observations, a non-empty subset of these k populations of preferably small size has to be selected, which contains the population with the largest mean with a probability of at least P^* at every parameter configuration. Several subset selection procedures which have been proposed in the literature are compared with Bayes selection procedures for normal priors under two natural types of loss functions. They considered two new subset selection procedures in this paper.

In [13], Gupta, Lin, and Lin investigated the problem of selecting the best logistic population from $k(\geq 2)$ possible candidates. The selected population must also be better than a given control. A selection procedure was developed. The performance (rate of convergence) of the proposed selection rule was analyzed as well. They also carried out a simulation study to investigate the rate of convergence of the proposed selection procedure. The results of simulation are provided in the paper.

Gupta and Liese [14] applied empirical Bayes method to construct selection rules for selecting all good exponential distributions. They modified the selection rule introduced and studied by Gupta and Liang who proved that the expected regret risk ER_n converges to zero with rate $O(n^{-\lambda/2}), 0 < \lambda \leq 2$. The aim of this paper was to prove a limit theorem for the random regret risk R_n. It is shown that nR_n tends in distribution to a linear combination of independent χ^2-distributed random variables. This result especially implies that under weak conditions the random regret risk is of order $O_p(\frac{1}{n})$.

Gupta, He and Li [16] investigated the problem of selecting the best population from (positive) exponential family distributions based on type-I censored data. A Bayes rule was derived and a monotone property of the Bayes selection rule was obtained. Following that property, they proposed an early selection rule. Through this early selection rule, one can terminate the experiment on a few populations early and possibly make the final decision before the censoring time. An example was provided in the final part to illustrate the use of the early selection rule for Weibull populations.

The research paper by Gupta and Li in [24] deals with the problem of selecting good ones compared with a control from $k(\geq 2)$ populations. The random variable associated with population π_i is assumed to be positive-valued and has density $f(x_i|\theta_i) = c(\theta_i)exp(-x_i/\theta_i)h(x_i)$ with unknown parameter θ_i, for each $i = 1, \cdots, k$. The distributions of parameters θ_i's are also unknown. A nonparametric empirical Bayes approach was used to construct the selection procedure. It was shown that the procedure is asymptotically optimal with a rate of $O(n^{-1})$. The results are applicable to data arising from (most) life-test experiments.

3.2. Test of Hypothesis $H_0 : \theta \leq \theta_0$ against $H_1 : \theta > \theta_0$.

In papers [15, 17, 21, 22, 23, 25], we have made contributions to research in this area.

In [15], Li and Gupta have considered the monotone empirical Bayes tests for a truncation parameter. A new approach was used to construct a monotone empirical Bayes test δ_n. An upper bound on the regrets of monotone empirical Bayes tests and an asymptotic minimax lower bound were obtained. Then the authors found the optimal rate of convergence of the tests. From the results established in this paper, it is concluded that the rule δ_n is optimal in the sense that (1) it has good performance for a small sample size since it possesses weak admissibility and (2) it has good performance for a large sample size since it possesses the optimal convergence rate.

Paper [17] studies monotone empirical Bayes tests for the normal mean under a linear loss. The optimal rate of convergence of the monotone empirical Bayes tests was obtained. Applying a few techniques and using the non-uniform estimate of the remainder in the central limit theorem, Li was able to construct a monotone empirical Bayes test and show that it achieves the best possible rate over a broad class of prior distributions. The best possible rate was found via a result of Donoho and Liu to construct the "hardest two-point subproblem". This answered the question raised by Karunamuni and Liang.

In [21], Gupta and Li considered empirical Bayes tests for testing $H_0 : \theta \leq \theta_0$ against $H_1 : \theta > \theta_0$ in the continuous one-parameter family with density $c(\theta)exp\{\theta x\}h(x)$, $-\infty \leq \alpha < x < \beta \leq \infty$ under the linear loss. Using the assumptions that $\int_\Omega |\theta| dG(\theta) < \infty$ and that the critical point b_0 of a Bayes test falls in some known interval $[C_1, C_2]$, where $\alpha < C_1 < C_2 < \beta$, they showed that, for any ε $(0 < \varepsilon < 1)$, the empirical Bayes tests can be constructed such that they have a convergence rate of order $o(n^{-1+\varepsilon})$, which generalizes the result of Liang(1999) from the positive (one-parameter) exponential family to any continuous one-parameter exponential family.

The problem investigated by Gupta and Li in [22] deals with empirical Bayes tests for testing $H_0 : \theta \leq \theta_0$ against $H_1 : \theta > \theta_0$ in the distribution family having density $f(x|\theta) = I_{[0 \leq x \leq \theta]} a(x)/A(\theta)$ under the linear loss. They constructed the empirical Bayes tests δ_n and $\overline{\delta}_n$ and proved that both of them have a good asymptotic property. As a special case, taking $a(x) = 1$ leads to a result obtained by Karunamuni (1999).

In paper [23], Gupta and Li considered empirical Bayes tests for testing $H_0 : \theta \leq \theta_0$ against $H_1 : \theta > \theta_0$ in some continuous exponential family with density $c(\theta)exp\{\theta x\}h(x)$, $-\infty \leq \alpha < x < \beta \leq \infty$, where $c(\theta)$ satisfies certain condition, under the linear loss. Using a natural assumption that $\int_\Omega |\theta| dG(\theta) < \infty$, they constructed the empirical Bayes test and showed that its regret goes to zero with a convergence rate of order $o(n^{-1+\frac{1}{\log\log n}})$. This rate of convergence improves the previous results regarding this problem in the sense that a much faster rate of convergence is achieved under weaker conditions. The applications of this result to $N(\theta, 1)$ and the general exponential family distributions were given as corollaries.

In [25], Gupta and Li considered the one-sided testing problem for lower truncation parameters through the empirical Bayes approach. The optimal rate of the monotone empirical Bayes tests was obtained and a monotone empirical Bayes test δ_n achieving the optimal rate was constructed. It was shown that δ_n has good performance for both small samples and large samples.

3.3. Other (Miscellaneous) Related Research.

Other research areas we have been involved in are design of experiments with selection and ranking goals, construction of lower confidence bounds for the PCS, tests of composite hypothesis, two-stage tests, two-stage procedures, and selection problems in measurement error models.

Gupta and Panchapakesan [2] have provided a review of design of experiments with selection and ranking goals. It covers mainly basic normal theory for single-factor experiments with and without blocking and 2-factorial experiments with and without interaction. They have referred to a few authors who have studied the problem using a Bayesian approach. They also discussed, as alternatives to tests of hypotheses among treatment means, three types of formulations: indifference-zone approach, subset selection approach and multiple comparison approach.

Gupta and Liang in [5] studied the problem of constructing simultaneous lower confidence bounds for the PCS_t, simultaneously, for $t = 1, \ldots, k-1$, for the general location-parameter models, where k is the number of populations involved in a selection problem and PCS_t denotes the probability of correctly selecting the t best populations. The result was then applied to the selection of the t best unknown means of normal populations.

In Paper [10], Liang studied the problem of testing $H_0 : \theta \in [\theta_1, \theta_2]$ versus $H_1 : \theta \notin [\theta_1, \theta_2]$ where $0 < \theta_1 < \theta_2 < \infty$, for the parameter θ in a discrete exponential family via the empirical Bayes approach. First, the behavior of the Bayes test was examined. Then the empirical Bayes test was constructed by mimicking the behavior of the Bayes test. The asymptotic optimality of the empirical Bayes test was investigated. It was shown that the proposed empirical Bayes test is asymptotically optimal and its associated Bayes risk converges to the minimum Bayes risk with a rate of order $O(exp(-cn))$ for some $c > 0$, where n is the number of historical data at hand for the present testing problem.

The empirical Bayes theory of two-stage tests for the two-action problem in a discrete exponential family was studied by Liang in [18]. An empirical Bayes two-stage test was constructed. Asymptotic optimality of the empirical Bayes two-stage test was investigated and the rate of convergence of its associated regret Bayes risk was established. It was shown that the proposed empirical Bayes two-stage test is asymptotically optimal with a rate of convergence of order $O(exp(-cn))$ for some $c > 0$, where n denotes the number of historical data at hand when the present decision problem is considered.

In [19], Gupta, Liang and Lin investigated the problem of selecting the treatment with the largest probability of success from $k(\geq 2)$ independent Bernoulli treatments. The data are taken using the inverse binomial sampling (i.e., the negative binomial models are considered). The selection treatment must be better than a given control. The authors employed the empirical Bayes approach and developed a two-stage selection procedure. They proved that the proposed selection rule is asymptotically optimal at the rate of convergence of order $O(\exp(-cn))$, for some positive c, where n is the number of the historical data at hand. A simulation study was also carried out to investigate the performance of the proposed empirical Bayes selection procedure for small to moderate values of n. The simulation results are provided in the paper.

Gupta and Lin [20] studied a selection problem for linear measurement error models A selection procedure was constructed and its asymptotic optimality was also investigated. It was shown that with the assumption of the existence of the α-th $(\alpha > 2)$

moment, the regret risk of the proposed selection procedure converges to zero with a rate of order $o(n^{-(\alpha/2-1)})$. It was further shown that the moment generating functions of the corresponding variables exist, the regret risk of the proposed selection procedure converges to zero with an order $O(e^{-cn})$ under mild conditions, where c is a positive constant.

4. Scientific Personnel.

Personnel supported by (involved in) this project are:

1. Professor Shanti S. Gupta, Purdue University: PI

2. Professor TaChen Liang, Wayne State University: Consultant

3. Professor Klaus J. Miescke, University of Illinois at Chicago: Consultant

4. Professor S. Panchapakesan. Southern Illinois University: Consultant

5. Professor Friedrich Liese, University of Rostock, Germany: Consultant

6. Dr. Xun Lin, Abbott Laboratories: received Ph.D. degree in April 1999. Student of Professor Gupta: Consultant

7. Mr. Lin Chen, Purdue University: Ph.D. student working with Professor Gupta

8. Mr. Jianjun Li, Purdue University: Ph.D. Student working with Professor Gupta

During this period, research has been done in the area of simultaneous inference and ranking and selection procedures with emphasis on Bayes and empirical Bayes procedures. Dr. Xun Lin completed the Ph.D. degree in April, 1999 under the direction of Professor Gupta.

5. Meetings attended and talks presented.

Professor Gupta gave an invited seminar lecture (Neyman Seminar) at the University of California, Berkeley in October 1995. He also gave an invited colloquium lecture at the University of California at Santa Barbara in November, 1995. In August 1995, he was an invited speaker at the 3rd International Chinese Statistical Association Conference in Beijing and later he participated in a special session on Bayesian Statistics at the 50th Session of the International Statistical Institute, also in Beijing.

In February, 1996, Professor Gupta was invited to visit three universities in Hong Kong. He gave four invited seminar talks: one at the Chinese University of Hong Kong, two at the University of Hong Kong and one at the Hong Kong University of Science and Technology. These lectures were related to single-stage and two-stage Bayes and empirical Bayes procedures. The talks were well received. Another event that took place was an interview for the Hong Kong Statistical Association Newsletter about the scientific developments in statistics and probability in the US universities. This interview was later published in the newsletter.

Professor Gupta also visited the University of Rome in April 1996 and gave an invited seminar talk at the Department of Statistics, Probability and Applied Statistics. He had interesting exchange of ideas about the Bayesian aspects of research going on there.

Professor Gupta was an invited participant at the 4th Bernoulli Society World Congress in Vienna on August 24-31, 1996. He gave an invited address at the conference on empirical Bayes simultaneous selection procedures for comparing normal populations with a standard. This paper was presented on August 26, 1996.

In 1997, Professor Gupta was invited to give talks at three international conferences. These are: International Symposium on Nonparametric Statistics and Related Topics, held at Carleton University, Ottawa, Canada, May 3-5, 1997; ISI-Satelite Meeting on Mathematical Statistics and its Applications to Bioscience, held at the University of Rostock, Rostock, Germany, August 31-September 4, 1997; International Conference on Recent Advances in Statistics and Probability held at the Indian Statistical Institute, Calcutta, India, December 29, 1997-January 1, 1998. These talks were related to single-stage Bayes and empirical Bayes procedures for simultaneous inference of selection and ranking. The meeting in Rostock was organized by the Institute of Mathematical Statistics and the European Regional Committee of the Bernoulli Society. The meeting in Calcutta was organized and sponsored by the Indian Statistical Institute and the Bernoulli Society for Mathematical Statistics and Probability.

In 1998, Professor Gupta was very heavily involved in the organization of the Sixth

Purdue International Symposium on Statistics, held on June 17 - 23, 1998. One part of this symposium was the Sixth Statistical Decision Theory Conference which was primarily organized by Professor Gupta. It was held on June 19 - 21, 1998. Several authors gave papers dealing with multiple decision theory and related topics at this conference. Two joint papers mentioned above (see [12] and [13]) were presented by the coauthors, Klaus Miescke and Xun Lin, respectively, at the symposium.

On October 10 and 11, 1998, Professor Gupta gave an invited talk at the International Indian Statistical Association Conference held at McMaster University, Hamilton, Ontario, Canada. This talk was concerned with the empirical Bayes selection procedures for exponential populations.

Professor Gupta gave an invited talk at the joint seminar of Eurandom and the Technical University of Eindhover, the Netherlands, in December, 1999. This talk was concerned with the empirical Bayes selection procedures for positive exponential family. Professor Gupta was invited to spend about two weeks at the European research named EURANDOM in the Netherlands. This visit resulted in fruitful exchange of ideas. Professor Gupta also gave a one-hour invited plenary session talk at the International Conference on "Statistics, Combinatorics and Related Areas", held at the University of South Alabama. Mobile, Alabama on December 18-20,1999. At this conference Mr. Jainjun Li also presented a talk on the empirical Bayes tests for some exponential family at this meeting.

6. Report of Inventions.

None.

7. Technology Transfer.

Nothing to report.